J. Fluid Mech. (1967), *vol.* 28, *part* 4, *pp.* 705–717
Printed in Great Britain

The crossed-beam correlation technique

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(Received 28 April 1966 and in revised form 6 September 1966)

This paper describes an experimental technique which allows estimates of local turbulent properties of a shear layer to be obtained without the necessity of inserting a probe into the flow field. The probe is replaced by two beams of radiation, which pass through the entire flow field in two mutually perpendicular directions. It is shown that, although each beam independently reflects only an integral of the flow properties along the entire path between the source and detector, the covariance between the two detected signals does yield local turbulent information.

To verify the validity of the technique, experimental results are presented for the shear layer of a subsonic jet and are shown to be in good agreement with published hot-wire data.

Introduction

The measurement of the turbulent properties of high-speed, supersonic shear layers has for some time represented an interesting challenge to the experimentalist. In addition, a growing awareness of the undesirable effects which these flows create on high-speed vehicles, particularly rocket launch vehicles, has increased the incentive to develop suitable measuring techniques.

To date, investigations of turbulent shear flows have been limited, almost exclusively, to the low-speed subsonic case. With the aid of hot-wire anemometer techniques, a considerable volume of data relating to the properties of such flows has become available. However, while the hot wire has proved to be an invaluable tool in these studies, in fact virtually the only one to find general adoption, its application has not been generally extended to the high-speed flows of more current interest. The reasons for this are well known, involving as they do problems of interpretation, creation of shock-wave disturbances by the probes, and the difficulties of using probe-type instruments in extreme temperature environments.

An awareness of the latter two problems in particular suggests the use of optical techniques. However, the disadvantage of the more standard optical techniques, such as schlieren, shadowgraph, or interferometry, lies in the fact that the measured quantity depends on an integral of the flow properties along the entire light path. This must normally extend across the entire flow field, while the technique required should give information on the local conditions existing at a chosen point within the flow field.

The purpose of this paper is to introduce a method of obtaining the required local information, while avoiding the necessity of introducing any instrumentation into the flow field. The technique involves the use of two beams of radiation which traverse the entire flow in two mutually perpendicular directions (figure 1). The radiation employed is arranged so that it is partially absorbed by a constituent of the flow. Thus, turbulence-induced fluctuations of either the thermodynamic properties or concentration of the chosen constituent are reflected as fluctuations of intensity of the resultant beam. Each beam alone, of course, reflects only an integral of the flow properties along its entire path. However, it is shown that correlation of the two resultant beam intensities eliminates much of this integration, yielding local turbulent information instead. It is shown that, using this method, local estimates of the turbulent intensity, eddy scales, spectra, convection speeds, and eddy lifetimes can be obtained.

§2 of this paper describes an experiment in which this technique was used for the determination of the turbulent properties of a subsonic jet exhaust. An acceptable degree of spatial resolution was obtained and the results are shown to be in agreement with published hot-wire data.

1. Description of the crossed-beam correlation method

The basic concept of the crossed-beam correlation technique can best be described with reference to figure 1. A region of turbulent flow is supposed to

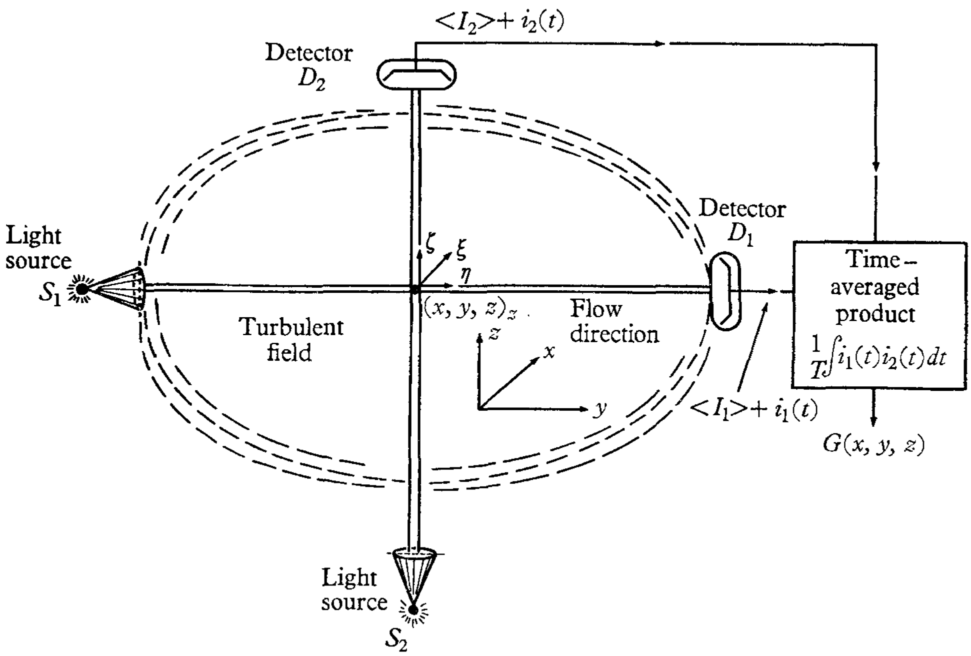

FIGURE 1. Schematic diagram of crossed-beam correlation principle.

be contained within the broken line, this flow being convected in a direction perpendicular to the plane of the diagram. Two optical systems are now arranged, which pass collimated beams of radiation across the flow in two mutually perpendicular directions so that they intersect at the point to be investigated. The wavelength of this radiation is arranged so that it is partially, but not completely, absorbed by one or more species of the flow. Thus turbulent fluctuations of the concentration or density of the chosen species will be reflected in changes of light intensity observed at the detectors. In common with the optical methods mentioned previously, each beam alone reflects only an integral of the appropriate fluctuation along its entire path length. However, as is shown below, the covariance existing between the signals at the two detectors does yield local information.

The retrieval of this local information can be explained intuitively as follows: the instantaneous signal at each detector represents the sum of all fluctuations occurring along its path at a particular time. The fluctuations can be categorized into two groups. First, those fluctuations which occurred sufficiently close to the beam intersection point to introduce a related (or correlated) fluctuation in both beams. The remainder which occur at a sufficient distance from this point are uncorrelated and thus introduce unrelated effects on the beam intensities. If, subsequently, the covariance (time-averaged product) of the two detected signals is estimated, those portions of the signal created by the unrelated (uncorrelated) flow fluctuations will yield an average value of zero. The related or correlated fluctuations, on the other hand, yield a finite averaged product. Thus, the measured covariance is a function only of those fluctuations which occur within the correlated area surrounding the beam intersection point. It remains, of course, to demonstrate that this covariance can be used as a measure of required turbulence parameters.

Analytical description

In order to demonstrate that the covariance of the two detected fluctuations does, in practice, yield a measure of required turbulent properties, it will be convenient to introduce the following co-ordinate system. The point of beam intersection has co-ordinates (x, y, z) where the y- and z-axes are oriented along the directions of the beams S_1D_1 and S_2D_2 respectively. Distances from the point of beam intersection are denoted by ξ, η and ζ in the x-, y- and z-directions, respectively.

Considering first the beam S_1D_1, the intensity recorded at detector D_1 at time t can be written

$$I_1(t) = I_0 \exp\left\{-\int K(x, y+\eta, z, t)\, d\eta\right\}, \tag{1}$$

where I_0 denotes the intensity of the initial beam and K is the appropriate extinction coefficient. The term 'extinction' is employed here to cover a number of possible methods of achieving the required beam attenuation. For example, pure absorption by a flow constituent could be employed, while scattering by particulate matter in the flow offers a second possibility. The term extinction is used here to cover either phenomenon or a combination of both. It should also be pointed out that the value of K will additionally be a function of the wavelength of the radiation employed. However, since this dependence does not affect the present discussion, it will not be explicitly shown.

Whatever the actual mechanism of extinction, it should obviously be chosen so that the value of the coefficient depends on a required flow property. Thus since, in a turbulent flow, the flow properties are a function of both position and time, the extinction coefficient will be similarly dependent. Throughout this paper, for the sake of generality, we shall refer to fluctuations of the extinction coefficient. However, since these changes will always reflect fluctuations of a flow property, statistical properties of the flow will be considered as synonymous with statistical properties of the extinction coefficient.

Returning to (1), we can write the instantaneous extinction coefficient as the sum of its time-averaged mean value $\langle K(x,y+\eta,z)\rangle$ and a fluctuation relative to this value $k(x,y+\eta,z,t)$. Then

$$I_1(t) = I_0 \exp\left\{-\int \langle K(x,y+\eta,z)\rangle d\eta\right\} \exp\left\{-\int k(x,y+\eta,z,t)\,d\eta\right\}. \tag{2}$$

If the extinction process is now arranged so that the integral of the fluctuations, i.e.

$$\int k(x,y+\eta,z,t)\,d\eta,$$

is sufficiently small to permit linearization of that portion of the exponential, (2) can be written

$$I_1(t) = I_0 \exp\left\{-\int \langle K(x,y+\eta,z)\rangle d\eta\right\}\left[1-\int k(x,y+\eta,z,t)\,d\eta\right]. \tag{3}$$

It should be made clear this is not an assumption which restricts the method to small fluctuations. First, the integral in question represents a sum of a number of statistically independent events, which will tend to reduce its value. Secondly, it is easily shown that, if the integral of the fluctuations is of order or less than 10 % of the mean integrated value, an optimum value for the mean attenuation is given by

$$\int \langle K(x,y+\eta,z)\rangle d\eta = 1.$$

For this magnitude of fluctuation the linearization would be acceptably accurate. In the event that larger fluctuations relative to the mean value are experienced, it would be both desirable and acceptable to reduce the mean absorption so that linearization is still possible.

If the signal at the detector is now written, in turn, as the sum of its time-averaged value $\langle I_1\rangle$ and a fluctuation $i_1(t)$ relative to this value, it is easily shown that

$$i_1(t) = -\langle I_1\rangle \int k(x,y+\eta,z,t)\,d\eta. \tag{4}$$

Thus, within the limits of the above discussion, we obtain the expected result that the fluctuation at the detector is proportional to the instantaneous integral of the fluctuations along the entire light path.†

† In the event that a situation arises in which fluctuations which are a high percentage of the mean value are to be measured and where a weak extinction process is not available, a result of the form of (4) can be obtained by introducing a logarithmic response amplifier at the output of the detector $i_1(t)$, then representing the output of this additional amplifier.

Considering next the beam S_2D_2, a similar result can be written down by inspection, namely

$$i_2(t) = -\langle I_2\rangle \int k(x, y, z+\zeta, t)\, d\zeta. \tag{4a}$$

If we now take these two fluctuating signals and measure their time-averaged product or covariance, we can define a quantity $G(x, y, z)$ where

$$G(x, y, z) \equiv \frac{1}{T}\int_0^T i_1(t)\, i_2(t)\, dt, \tag{5}$$

where T denotes a period of integration, which is of sufficient length to yield a statistically stationary value of $G(x, y, z)$. Substituting for $i_1(t)$ and $i_2(t)$ from (4) and (4a) respectively and reversing the order of spatial and temporal integration, (5) can be written

$$G(x, y, z) = \langle I_1\rangle\langle I_2\rangle \int_\eta\int_\zeta \frac{1}{T}\int_0^T k(x, y+\eta, z, t)\, k(x, y, z+\zeta, t)\, dt\, d\zeta\, d\eta. \tag{6}$$

To summarize, taking the fluctuating portions of the two detected signals and measuring their covariance, we obtain the result represented by (6).

Spatial resolution of covariances and mean-square values

We can most conveniently understand the significance of this result by considering initially the temporal integration, i.e.

$$\frac{1}{T}\int_0^T k(x, y+\eta, z, t)\, k(x, y, z+\zeta, t)\, dt \tag{7}$$

alone. This term clearly represents the covariance of the fluctuations at the points $(x, y+\eta, z, t)$ and $(x, y, z+\zeta, t)$. If one or both of these points are sufficiently far from the beam intersection point, the fluctuations will be mutually random and the resulting covariance will be zero. In fact, only those points contained within the correlated area around the beam intersection point will contribute to the measured value of $G(x, y, z)$. Thus, although formally the limits of spatial integration in (6) extend from source to detector the value of $G(x, y, z)$ is not changed if these limits are replaced by those corresponding to the limits of the locally correlated area. In this way, therefore, the measured quantity reflects only local turbulent information.

Rewriting the covariance in (6) as the product of the r.m.s. intensities at the points considered and a space correlation coefficient $R(x, y+\eta, z+\zeta)$, we obtain†

$$G(x, y, z) = \langle I_1\rangle\langle I_2\rangle \int_\eta\int_\zeta \{\overline{k^2(x, y+\eta, z, t)}\;\overline{k^2(x, y, z+\zeta, t)}\}^{\frac{1}{2}} R(x, y+\eta, z+\zeta)\, d\zeta\, d\eta. \tag{8}$$

If the intensity does not vary appreciably over the correlated area (i.e. the region for which the correlation coefficient is finite) then the measured quantity, $G(x, y, z)$, is proportional to this intensity and an area which, by analogy to the familiar concept of an integral length scale, we shall term the integral correlation area.

† Overbars denote time-averaged values.

In the more likely event that the intensity does vary, $G(x, y, z)$ then represents the product of the integral correlation area and an average of the intensities contained in that area. However, owing to the fact that the space correlation coefficient, $R(x, y+\eta, z+\zeta)$ will decrease as the separation of the points considered increases, this average is heavily weighted towards the values close to the beam intersection point.

Thus, to a useful degree of approximation we can rewrite (8) in the form

$$G(x,y,z) = \langle I_1 \rangle \langle I_2 \rangle \overline{k^2(x,y,z,t)}\, L_y L_z, \tag{9}$$

where L_y and L_z are the integral† length scales in the y- and z-directions respectively.

The measured quantity therefore reflects directly the local turbulent intensity at the point of beam intersection.

Two-point space–time correlations

In order to demonstrate other turbulent properties which can be measured using this technique let us consider the result obtained when one beam is displaced a distance ξ in the streamwise direction, while in addition a time delay, τ, is introduced between the detected signals prior to estimating their time-averaged cross-product. This situation is shown schematically in figure 2. Denoting the resulting cross-correlation by $G(x+\xi, y, z, \tau)$ the result can be written down by inspection of (6),

$$G(x+\xi,y,z,\tau) = \langle I_1 \rangle \langle I_2 \rangle \int_\eta \int_\zeta \frac{1}{T} \int_0^T k(x,y+\eta,z,t)\, k(x+\xi,y,z+\zeta,t+\tau)\, dt\, d\zeta\, d\eta. \tag{10}$$

The interpretation of any one term within the double space integral is again straightforward. It represents the space–time covariance for the points $(x, y+\eta, z)$ and $(x+\xi, y, z+\zeta)$ for the value of time delay τ. Therefore finite contributions to the double space integral will be obtained only from those pairs of points which experience a common fluctuation. These points will be restricted to a local region around the line of minimum beam separation, where a fluctuation incident on the upstream beam subsequently passes through the downstream beam.

Rewriting (10) in a form similar to (8) we obtain

$$\begin{aligned} G(x+\xi,y,z,\tau) = \langle I_1 \rangle \langle I_2 \rangle \int_\eta \int_\zeta \{\overline{k^2(x,y+\eta,z,t)}\, \overline{k^2(x+\xi,y,z+\zeta,t)}\}^{\frac{1}{2}} \\ \times R(x+\xi, y+\eta, z+\zeta, \tau)\, d\zeta\, d\eta, \end{aligned} \tag{11}$$

where $R(x+\zeta, y+\eta, z+\zeta, \tau)$ is the appropriate space–time correlation coefficient.

If it is assumed that over the local range of η and ζ for which this correlation coefficient is finite the convective flow properties are relatively independent of η and ζ we can use a separation of variable assumption to obtain

$$R(x+\xi, y+\eta, z+\zeta, \tau) = R(x, y+\eta, z+\zeta)\, r(\xi, \tau). \tag{12}$$

† The authors are grateful to a reviewer for pointing out that in obtaining (9) from (8) an unnecessary assumption has been included; namely that the function $R(x, y+\eta, z+\zeta)$ is separable. If such an assumption is not justified the true double integral of this quantity still represents an area which merely determines the constant of proportionality between G and $\overline{k^2}$.

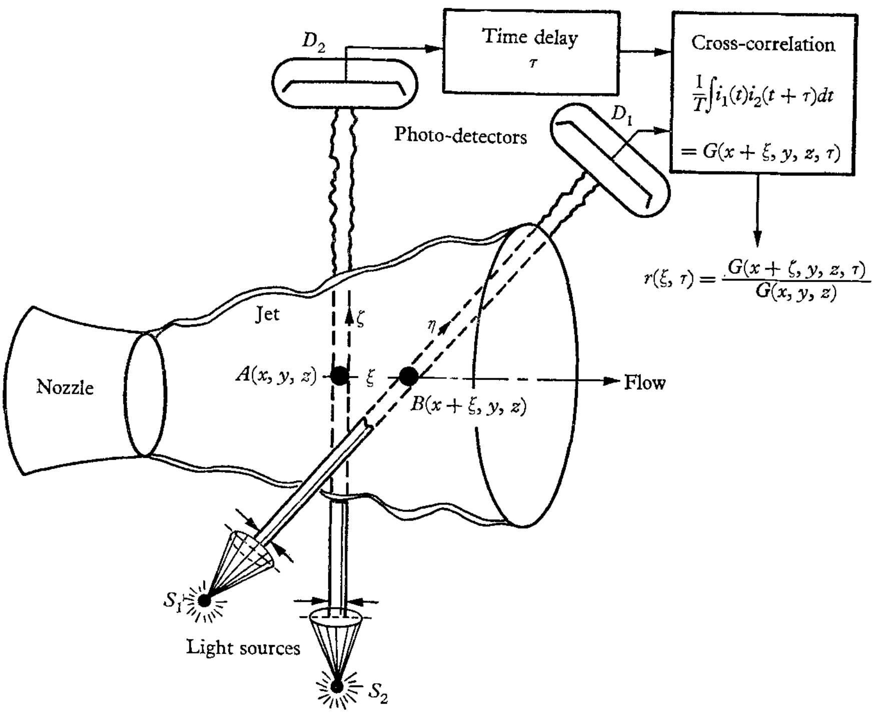

FIGURE 2. Schematic diagram of crossed-beam operation with downstream beam separation.

Here $r(\xi, \tau)$ is the space–time correlation coefficient which would be measured by two point probes located at the points A and B respectively in figure 2. $R(x, y+\eta, z+\zeta)$ is a weighting factor which decreases as the value of η or ζ increases.

Substituting expression (12) into (11) the measured cross-correlation becomes

$$G(x+\xi, y, z, \tau) = r(\xi, \tau)\langle I_1\rangle\langle I_2\rangle \int_\eta \int_\zeta \{\overline{k^2(x, y+\eta, z, t)}\,\overline{k^2(x+\xi, y, z+\zeta, t)}\}^{\frac{1}{2}} \times R(x, y+\eta, z+\zeta)\,d\zeta\,d\eta. \quad (13)$$

A comparison with (8) indicates that, to a useful degree of approximation $r(\xi, \tau)$, the required space–time correlation coefficient, is the ratio of two measurable quantities, i.e.

$$r(\xi, \tau) = G(x+\xi, y, z, \tau)/G(x, y, z). \quad (14)$$

If the space–time correlation coefficient is measured over a representative range of both ξ and τ the following properties of the turbulent flow field can be obtained.

(*a*) The space correlation coefficient $r(\xi, 0)$ from which the integral scale of turbulence is obtained by integration over all ξ.

(*b*) The auto-correlation coefficient, $r(0, \tau)$, which yields the turbulent spectrum by Fourier transformation.

(*c*) The velocity of convection obtained from the time delay at which a particular cross-correlation curve exhibits a maximum value.

(*d*) The moving axes auto-correlation, which is the envelope of a series of such cross-correlation curves.

(*e*) The eddy lifetime, which may be defined as the time delay for which the moving axes auto-correlation falls to $1/e$ of its initial value.

Thus, it appears in principle that many of the properties which have been previously measured in subsonic flows using hot-wire anemometer techniques can now be measured using the crossed-beam correlation method. This method eliminates the necessity of inserting solid probes into the flow field, thereby permitting measurements to be made over a wider range of turbulent flows than has been possible in the past.

Summary

The purpose of § 1 of this paper has been to discuss the basic concepts involved in the optical crossed-beam correlation method. It has been shown that a combination of absorption measurements with cross-correlation analysis can be used to obtain estimates of local turbulent flow properties.

Equation (9) indicates that the measured covariance is directly proportional to the turbulent intensity at the beam intersection point. With the assumption that the integral length scales L_y and L_z are not strong functions of position for a given cross-section of the flow, a relative turbulent intensity profile can be obtained directly by measuring this covariance for a series of beam intersection points. Obviously, the measured quantity is not strictly a point value, but represents a weighted average of the intensity over the locally correlated area. However, a calculation in which empirical expressions were developed for the intensity profile and lateral space correlations as measured in a subsonic jet (Davies & Fisher 1963) shows that the weighting is such that the measured quantity follows the true intensity profile to an accuracy of order 5 % over the entire cross-section of the flow.

To obtain the absolute level of the fluctuations of a flow property the product $L_y L_z$ must necessarily be known, as well as the relationship between this property and the extinction coefficient. The latter can conveniently be obtained from calibration measurements using a standard absorption cell. Although in principle the relevant length scales can be obtained, their experimental determination strictly requires one beam to be orientated along the flow direction. This may prove troublesome in practice, but it is felt that useful estimates of the length scales can be obtained with a practical degree of beam reorientation.

By contrast the kinematic flow properties are more easily obtained. The integral length scale in the flow direction, the turbulent spectrum, convection velocity, moving-axes time scale and eddy lifetime can all be obtained from the space–time correlation coefficient $r(\xi, \tau)$ which is given by (14)

$$r(\xi, \tau) = G(x+\xi, y, z, \tau)/G(x, y, z).$$

Thus to obtain these parameters only the ratios of measured cross-correlations are required.

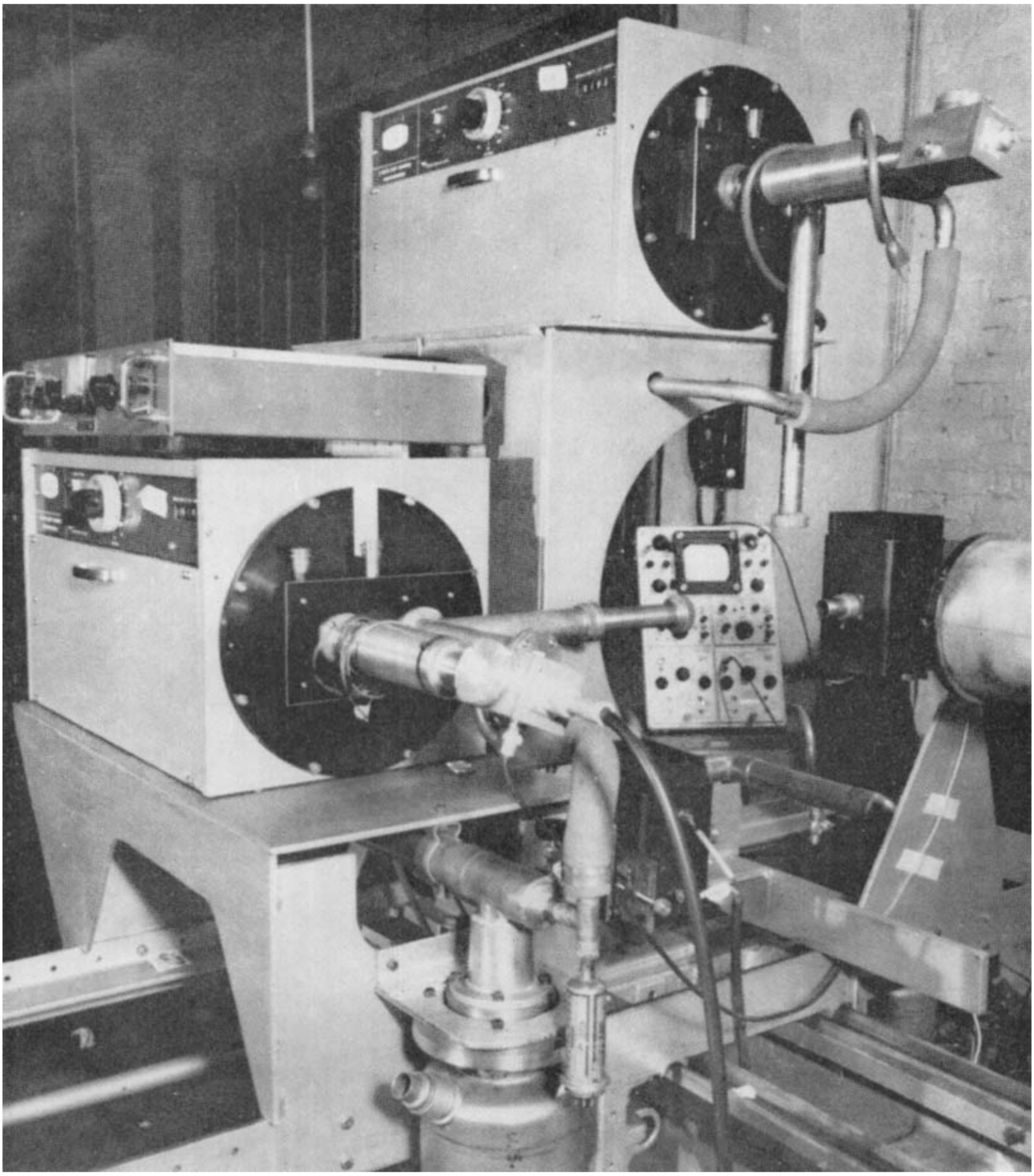

FIGURE 3. Crossed-beam correlation apparatus.

2. Experimental results

In order to demonstrate the feasibility of the crossed-beam correlation technique an experiment was performed in the exhaust of a 1 in. diameter model subsonic jet. This experiment has the advantage that the properties of the flow have been established using hot-wire anemometer techniques. These results are well documented in the literature and direct comparison with the optical method is therefore possible.

All measurements reported here were taken in a subsonic ($M = 0{\cdot}2$) air jet exhausting to atmosphere. Light extinction was achieved by injecting a water mist tracer into the flow, which attenuated the light beams by scattering. Changes in detected light intensity, therefore, reflect fluctuations of local tracer concentration.

The apparatus

A single arm of the optical system for this experiment comprised basically a powerful mercury arc lamp and a collecting system which passes a collimated beam of radiation across the flow field. This beam, after passage across the flow, was passed through a monochromator, the transmitted radiation subsequently being detected by a photo-multiplier. In order to obtain adequate wave-number resolution of the turbulent fluctuations, an aperture stop was employed in the receiving system which limited the effective beam diameter to 2 mm.

Two similar systems were arranged, one producing a light beam in the horizontal direction, the other in a vertical plane. The complete apparatus was mounted on a lathe bed to provide a rigid base for mechanical support and stability. The horizontal beam passed through the jet diameter and could be located at any required axial station. The vertical beam could be moved independently in either an axial or a radial direction, thus permitting the beam intersection point to be varied as required. A view of the apparatus is shown in figure 3, plate 1.

Results

Typical sets of results are shown in figures 4 and 5 respectively. The results of figure 4 were obtained with the beams initially arranged to intersect three diameters downstream from the lip of the jet, half a diameter from the jet axis. The vertical beam was subsequently displaced downstream in discrete steps. The separation employed is indicated on each correlation curve. At each station magnetic tape recordings were made of the detected fluctuations. These sample recordings were processed using the digital data reduction routine RAVAN (Random Vibration Analysis). This was developed by the George C. Marshall Space Flight Center to process signals and yield correlation data. It is seen from figure 4 that the presence of turbulent convection is clearly indicated by the displaced maxima of the correlation curves, while the decrease in amplitude of these maxima shows the decay of the turbulent pattern. The time delay at which the moving axes auto-correlation falls to a value $1/e$ is estimated from figure 4 to be $880\,\mu$s. This is in good agreement with the hot-wire data of Davies, Fisher & Barratt (1963), from which, for our experimental conditions, we estimate a value of $920\,\mu$s.

Figure 5 shows the results of a similar experiment performed six diameters from the jet exit at the same radial location. Unfortunately, the extent of streamwise beam separation employed here was not sufficient to permit the direct measurement of the moving-axis time scale. However, for the purposes of comparison a moving-axes auto-correlation, calculated on the basis of the results of Davies *et al.* (1963) is shown as the dotted line superimposed on the measured cross-correlation curves. Once again, satisfactory agreement is indicated.

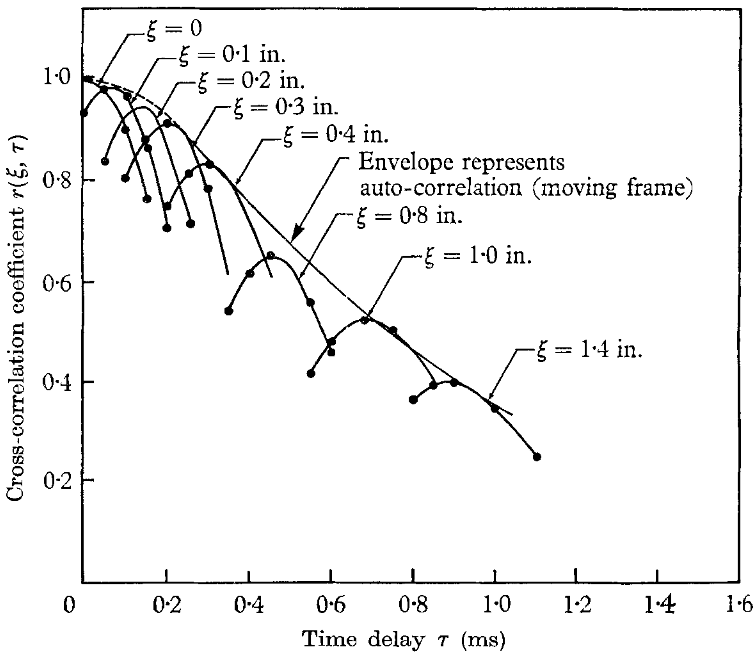

FIGURE 4. Cross-correlation with downstream beam separation. Jet exit velocity = 245 ft./s. $X/D = 3{\cdot}0$, $Y/D = 0{\cdot}5$.

The upper portion of figure 5 shows a plot of beam separation against the time delay at which a particular cross-correlation curve touches the moving-axes auto-correlation. The slope of the resulting line indicates the convection velocity.

Convection velocity measurements have been made for a range of radial positions corresponding to the axial locations of figures 4 and 5. A summary of these results is presented in figure 6, where they are compared with the hot-wire determinations of Davies *et al.* (1963). The mean velocity profile as reported by Davies *et al.* (1963) is also shown for comparison.

In the outer portion of the shear layer ($\eta \geqslant 0$) the agreement between the hot-wire and crossed-beam results is within experimental error. For $\eta < 0$, however, the crossed-beam results appear to follow the mean velocity profile (U/U_0) rather than attain the constant maximum value ($U_c/U_0 \sim 0{\cdot}7$) observed for the hot-wire data. This result was not unexpected. All convection velocity results presented by Davies *et al.* (1963) were taken at axial stations where the potential core still exists. At these positions the large discrepancy between the mean velocity and convection velocity has never been completely explained. Davies (1965) has recently discussed the subject and has attributed the effect to the fact that, in the potential core, the hot wire detects near field pressure fluctuations generated in the nearby shear layer. Thus, the fluctuations tend to travel downstream at a convection velocity corresponding to that at the generation point rather than at

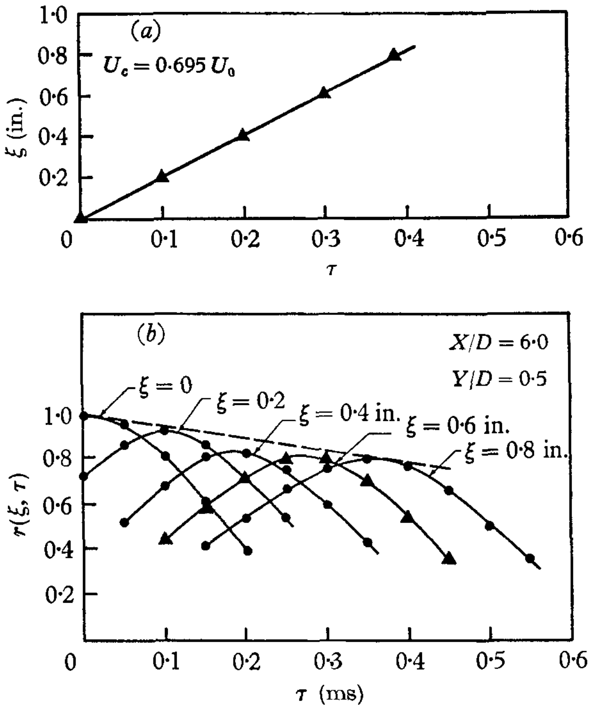

FIGURE 5. Cross-correlation with downstream beam separation (convection velocity from upper figure). Jet exit velocity $U_0 = 245$ ft./s.

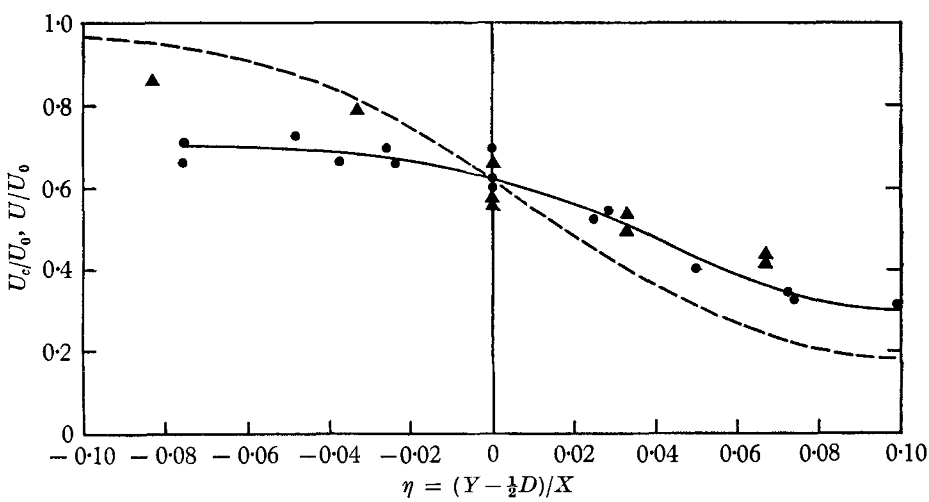

FIGURE 6. Radial distribution of convection velocity (hot-wire and cross-beam methods). ●, hot wire; ▲, cross-beam; —, U_c/U_0; ---, U/U_0.

the local flow velocity. It does appear reasonable, therefore, to expect the closer correspondence between the convection velocity and mean velocity when a measuring technique other than the hot-wire anemometer is used and where the potential core no longer exists. The two results in question were, in fact, both obtained at an axial location six diameters from the jet where the potential core has long since disappeared.

The importance of the results presented in figure 6 lies in the fact that an acceptable variation of convection velocity across the shear layer has been obtained. This does indicate that a useful degree of spatial resolution of turbulent properties can be obtained using this technique.

Conclusions

The primary purpose of this paper has been to introduce the basic concepts and approximations involved in using the crossed-beam correlation technique for determining local turbulent flow properties, while avoiding the necessity of inserting probes into the flow field. § 1 discusses the various measured quantities which can be combined to yield local estimates of the turbulent intensity, eddy scales, turbulent spectra, convection velocity and eddy lifetimes. § 2 presents some preliminary experimental results measured in the exhaust of a subsonic model jet. Satisfactory agreement with hot-wire data is obtained, while the convection velocity profile, presented in figure 6, indicates that a useful degree of spatial resolution is obtainable.

Although the experiment reported here was performed in a low-speed flow, using a tracer to obtain the required fluctuations of light intensity, it is to be emphasized that the main utility of the method is in the investigation of high-speed flows where instruments located in the flow field cannot be used. In addition, the use of a tracer is not, in principle, a necessity. The radiation to be absorbed can be chosen from any portion of the electromagnetic spectrum. Thus, in many situations the necessary beam attenuation can be created by a species naturally present in the flow. Fluctuations of light intensity then reflect variations of the thermodynamic properties or concentration of the chosen species. In multi-component or reacting flows a suitable choice of the wavelength and wavelength interval of the radiation could be employed to monitor fluctuations of a particular component of interest.

Therefore, not only does this method permit the measurement of local turbulent properties while avoiding the necessity of disturbing the flow field with the measuring instrument; but, in addition, control of the wavelength of the radiation allows considerable flexibility, not available with conventional turbulence measurement techniques.

This work was supported in part under Contract NAS8-11258, and represents a programme of work undertaken jointly by the IIT Research Institute and the George C. Marshall Space Flight Center.

The authors wish to acknowledge the contributions of the following personnel of the IIT Research Institute: Dr A. J. Montgomery and Mr A. Appel for the design and construction of the optical system; Dr T. Black, Mr J. M. Clinch, Mr D. W. Prosser, Mr J. Fitzgerald and Mr S. Pernic for their assistance in the experimental work and data analysis.

The provision of computer programs and the running of the statistical data reduction by Mr J. Jones and Mr M. Newberry, R.Comp.R.R.V., at the George C. Marshall Space Flight Center, is also gratefully acknowledged.

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